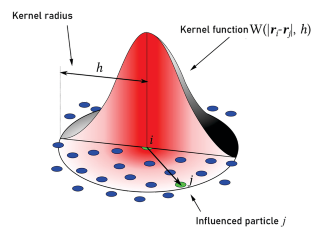
SMOOTHED PARTICLE HYDRODYNAMICS & MAGNETOHYDRODYNAMICS

Smoothed Particle Hydrodynamics (SPH)

SPH is a particle-based simulation method. It represents fluids as a collection of discrete particles; each particle carries properties like mass, position, velocity, and thermodynamic quantities.

SPH is a Lagrangian method, meaning we follow particles over time rather than evaluating them on a fixed grid. The fluid properties at any point in space are obtained by averaging over the properties of nearby particles, with a smoothing kernel function that weights the contribution of each particle based on its distance.



How the parameters are set for the smoothing kernel function defines how the fluid will behave. Common kernel functions include the cubic spline and Gaussian kernels, each with specific properties that influence how particles interact. The width of the kernel (defined by the smoothing length h) determines the range of interaction, allowing SPH to naturally adapt resolution based on particle distribution.

Any continuous property of the fluid at the position $\mathbf{r}$ can be calculated by summing the contributions from neighboring particles, using the smoothing kernel function to weight these contributions based on distance.

$$f(\mathbf{r}) \approx \sum_j m_j \frac{f_j}{\rho_j} W(\mathbf{r}-\mathbf{r}_j, h)$$

where m_j is the mass of the j-th particle, ρ_j is the density of the j-th particle, f_j is the value of the function at the position of the j-th particle, $W(\mathbf{r}-\mathbf{r}_j, h)$ is the smoothing kernel function, defining the volume over which particles interact, and h is the smoothing length.

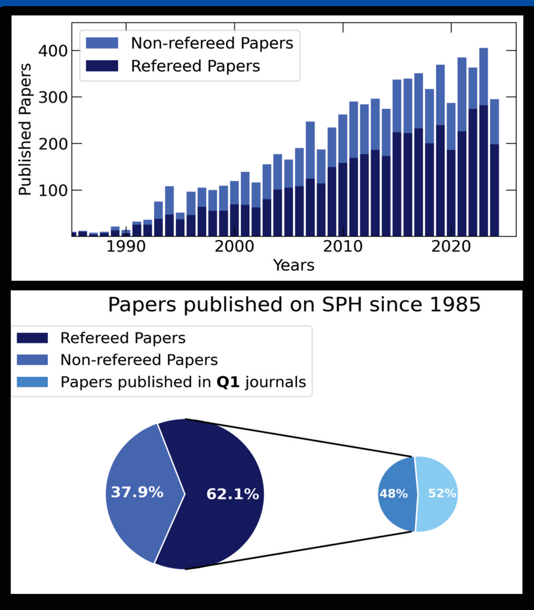
Each particle's mass is fixed, but its density is computed dynamically based on the proximity of neighboring particles:

$$\rho_i = \sum_j m_j W(\mathbf{r}_i - \mathbf{r}_j, h)$$

SPH inherently conserves mass, momentum, and energy as a direct result of its particle-based nature and the way interactions are symmetrically treated between neighboring particles. Fluid motion is governed by the Navier-Stokes momentum equation approximated for SPH methods in a particle-based form:

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right) \nabla_i W(\mathbf{r}_i - \mathbf{r}_j, h)$$

where $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$ are the pressures at particles i and j, and $\boldsymbol{v_i}$ is the velocity of particle i.



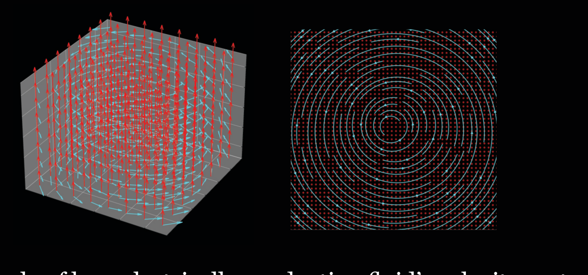
Smoothed Particle Hydrodynamics (SPH), first introduced in the 1977 by Gingold and Monaghan and Lucy, has become a powerful method for simulating complex astrophysical systems. It is used in simulations spanning scales from the formation of stars, just a few solar radii across, to large-scale galaxy mergers, extending over hundreds of thousands of light-years.

SPH is used in areas of astrophysics and engineering, where typical simulations involve millions of particles to resolve fluid interactions. In some state-of-the-art simulations, SPH models can reach hundreds of millions of particles to capture detailed processes like gas dynamics in planetary formation or the collapse of molecular clouds.

Thanks to improvements in computational power, SPH simulations are becoming more and more available, allowing researchers to explore previously unreachable regimes of fluid dynamics. This has led to over 1,000 publications in high-impact astrophysical journals like A&A and ApJ in the last decade.

Magnetohydrodynamics (MHD)

MHD is the study of the dynamics of electrically conducting fluids, like plasmas or liquid metals. It combines the principles of fluid dynamics and electromagnetism by extending Navier-Stokes equations to include the effects of magnetic fields, coupled with Maxwell's equations to describe how those magnetic fields evolve over time.



Basic example of how electrically conducting fluid's velocity vector and its magnetic field are aligned. Red arrows show the field's direction, cyan arrows - fluid's motion direction.

The basic idea is that in these fluids, the movement of charged particles creates electric currents, which generate magnetic fields. These magnetic fields then interact with the fluid, affecting its motion and creating forces that influence the flow, creating a feedback loop. This interaction leads to complex effects, like magnetic waves and plasma confinement.

The ideal MHD equations consist of a set of coupled partial differential equations for the fluid and the magnetic field. They combine the Navier-Stokes equations of fluid dynamics with Maxwell's equations:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{v}) = 0 \quad \text{– the continuity equation,}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{\rho}\boldsymbol{\nabla}P - \frac{1}{4\pi\rho}(\boldsymbol{B}\times \mathbf{rot}\,\boldsymbol{B}) + \boldsymbol{f} \quad \text{– the motion equation,}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{rot}(\boldsymbol{v}\times\boldsymbol{B}) \quad \text{– the induction equation,}$$

$$\frac{\partial \varepsilon}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\varepsilon = -\frac{P}{\rho}\boldsymbol{\nabla}\cdot\boldsymbol{v} \quad \text{– the energy equation.}$$

To close the set an equation of state $P = P(\rho, s)$ must be added. Additionally, magnetic field must satisfy the solenoidal condition: $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$.

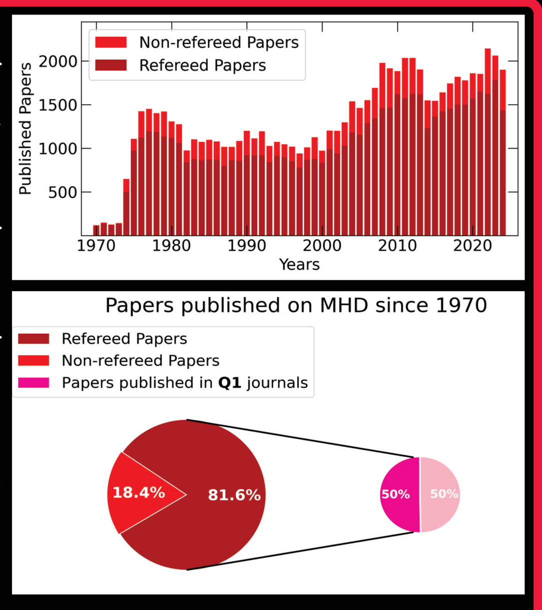
The magnetic viscosity must be taken into account in the induction equation when considering magnetic field diffusion:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{rot}(\boldsymbol{v}\times\boldsymbol{B}) - \mathbf{rot}(\eta\,\mathbf{rot}\,\boldsymbol{B}).$$

Then, a heating source due to current dissipation (Joule-Lenz effect) needs to be included in the energy equation:

$$\rho\left[\frac{\partial \varepsilon}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\varepsilon\right] + P\boldsymbol{\nabla}\cdot\boldsymbol{v} = \frac{\eta}{4\pi}(\mathbf{rot}\,\boldsymbol{B})^2.$$

The theory of magnetohydrodynamics (MHD) for electrically conducting fluids was developed by Hannes Alfvén in his 1942 paper, "Existence of Electromagnetic–Hydrodynamic Waves," published in Nature. In this work, he described the discovery of "electromagnetic–hydrodynamic waves", that are now known as Alfvén waves. In 1970 Alfvén was awarded The Nobel Prize in Physics for this work.

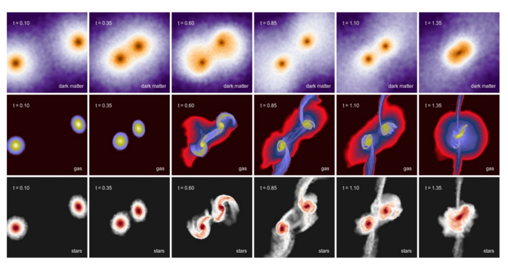


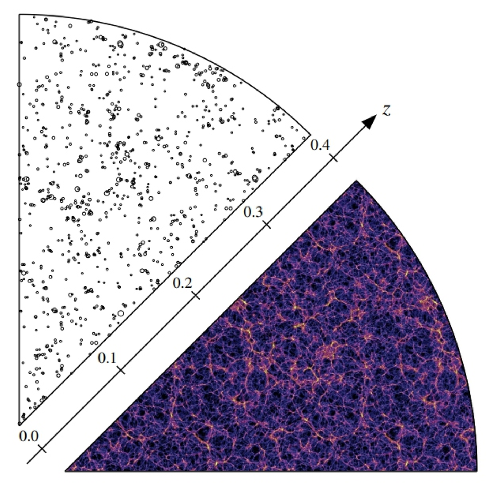
MHD models have been used to simulate wide range of phenomena from physics of semi-conductors on nanometre scale to galactic spiral arms reaching thousands of light-years into space.

MHD simulations allow modeling complex physical systems, such as the multi-million-degree plasmas in solar flares, explaining magnetic reconnection events that release over 10^25 joules of energy. Another example of MHD use are accretion disks around compact objects like black holes, where magnetic fields control matter flows and generate powerful jets.

Implementations in Astrophysics

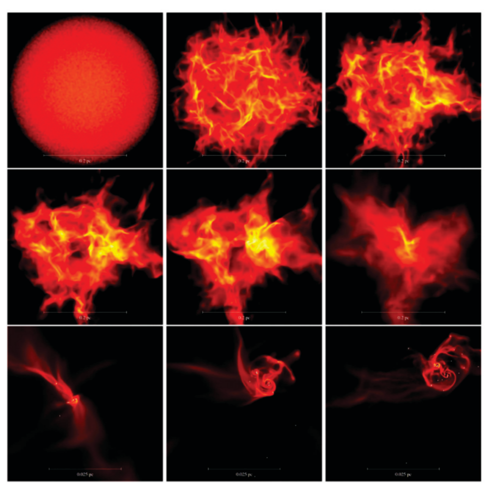
GADGET: N-body and SPH code designed for cosmological simulations that can model galaxies formation, evolution and dynamics.

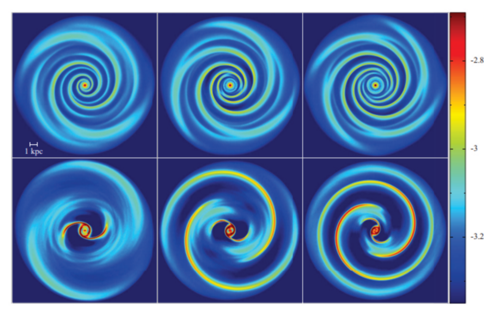




Springel V., Pakmor R., Zier O., Reinecke M., "Simulating cosmic structure formation with the GADGET-4 code", MNRAS, submitted

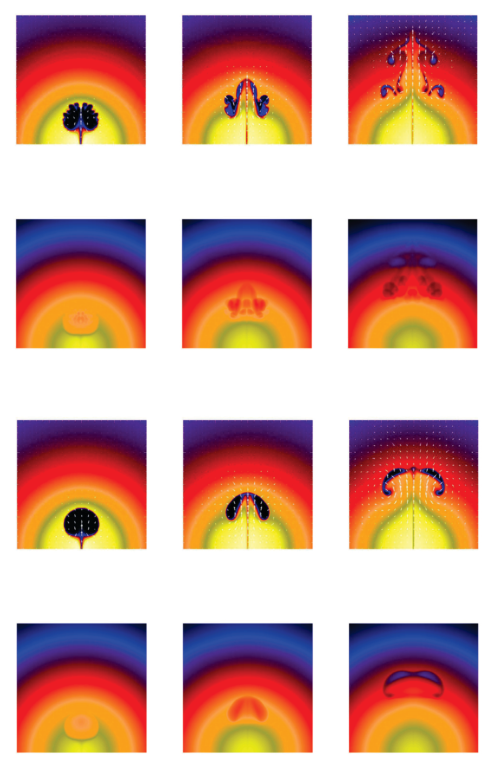
PHANTOM: A modular, user-friendly SPH & MHD code designed to simulate wide range of astrophysical phenomena, from protoplanetary disks to galactic dynamics and high-energy astrophysics.





Price, D. J., Wurster, J., et al. (2018). Phantom: A Smoothed Particle Hydrodynamics and Magnetohydrodynamics Code for Astrophysics. Publications of the Astronomical Society of Australia, 35, e031.

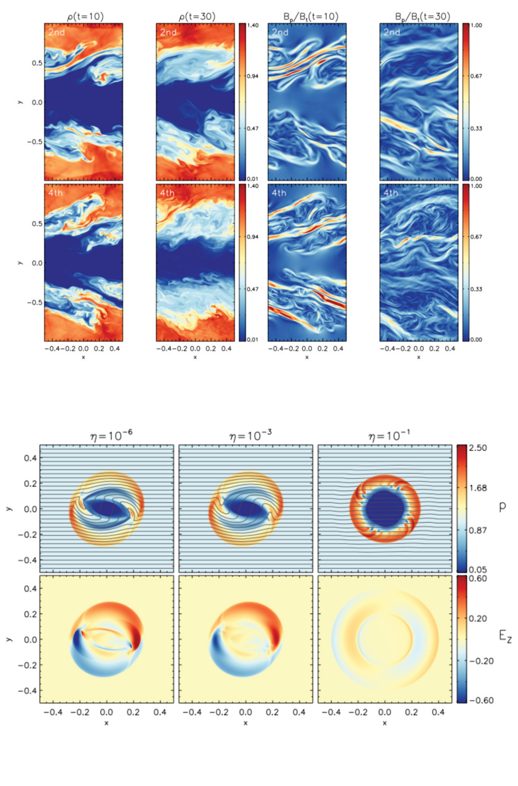
ZEUS-MP: A finite-difference MHD code for simulating astrophysical plasmas. It's used in accretion disk modeling, supernova remnants, and magnetic reconnection in solar and stellar environments.



"Buoyant radio-lobes in a viscous intracluster medium" C.S.Reynolds, B.McKernan, A.C.Fabian, J.M.Stone, J.C.Vernaleo, 2005, MNRAS, 357,242.

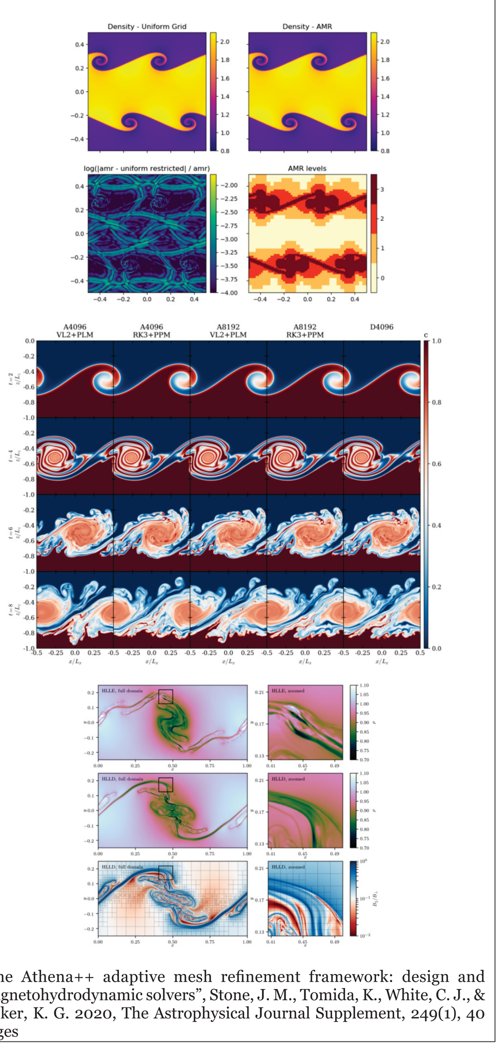
PLUTO: A versatile MHD code for solving fluid equations for high-speed and high-energy flows in space plasma.

It uses Godunov-type methods to handle shock waves, and calculates using either finite volume or finite difference.



4th-Order Finite Volume method paper (resistive RMHD): "A fourth-order accurate finite volume scheme for resistive relativistic MHD", Mignone et al., MNRAS (2024) 533, Issue 2, pp.1670-1686.

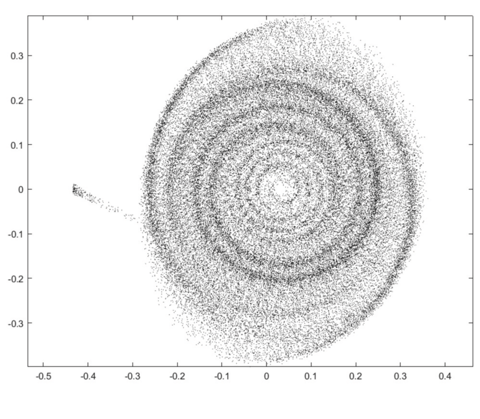
ATHENA++: MHD code that uses a finite-volume method to solve the fluid and MHD equations on either fixed or adaptive grid.



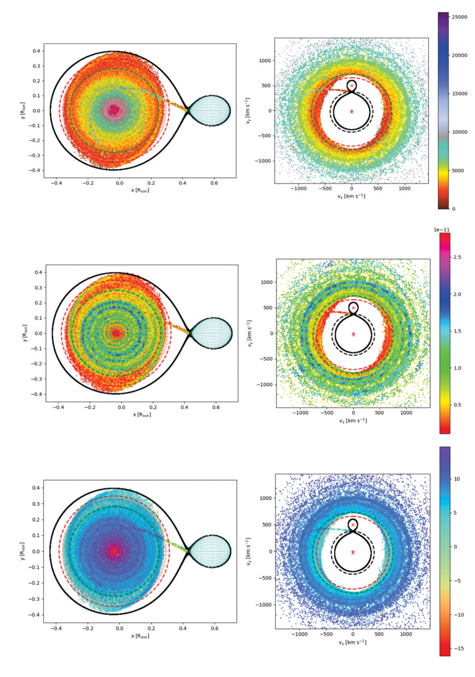
"The Athena++ adaptive mesh refinement framework: design and magnetohydrodynamic solvers", Stone, J. M., Tomida, K., White, C. J., & Felker, K. G. 2020, The Astrophysical Journal Supplement, 249(1), 40 pages

OUR SPH MODELS

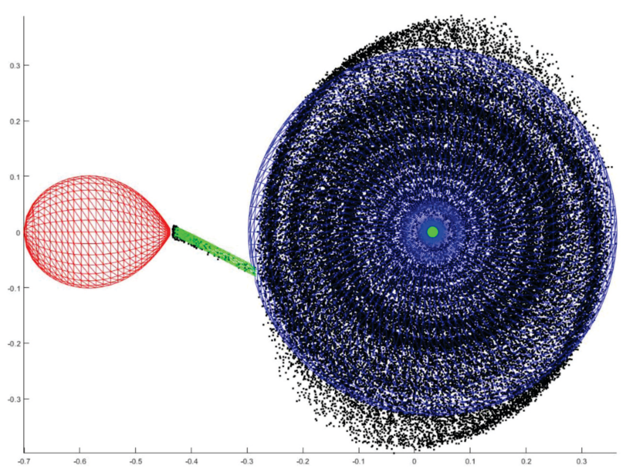
Below are different implementations of the SPH modeling as described in [1] of the accretion disk of a cataclysmic variable EZ Lyn. It is a cataclysmic variable with a 0.85 M⊙ white dwarf and 0.048 M⊙ brown dwarf with an orbital period of 5079.6 seconds. The system demonstrates the presence of spiral patterns in the outer rim of the disk caused by the 2:1 resonance [2]. 2:1 and 3:1 resonance radii are marked as dashed circles on our models.



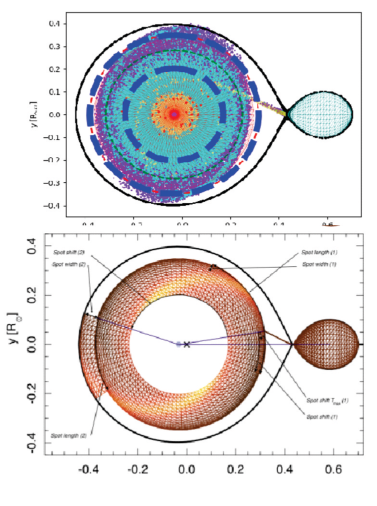
Particles' distribution in the accretion disk (in spatial coordinates).

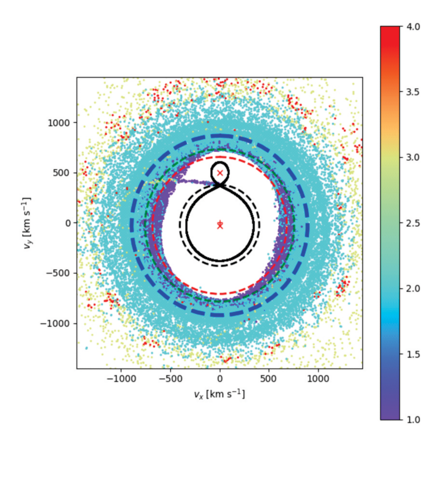


Temperature (top panel), density (middle panel) and intensity (bottom panel) distributions in the accretion disk. Left panels are in spatial coordinates and right panels are in velocity coordinates.



Example of combining SPH model with CVLab model (light curve-based modelling).





Domination of radiation types in accretion disk.

1. Liu, M. B., Liu, G. R., & Lam, K. Y. (2003). Constructing smoothing functions in smoothed particle hydrodynamics with applications. Journal of Computational and Applied Mathematics, 155(2), 263–284.

2. Amantayeva, A., Zharikov, S., Page, K. L., Pavlenko, E., Sosnovskij, A., Khokhlov, S., & Ibraimov, M. (2021). Period Bouncer Cataclysmic Variable EZ Lyn in Quiescence. The Astrophysical Journal, 918(2), 58.

3. Bisikalo, D., Kaygorodov, P., Ionov, D., Shematovich, V., Lammer, H., & Fossati, L. (2013). Three-dimensional Gas Dynamic Simulation of the Interaction between the Exoplanet WASP-12b and its Host Star. The Astrophysical Journal, 764(1), 19.

4. Seaton, M. J., et al. "Opacities for stellar envelopes." *Monthly Notices of the Royal Astronomical Society* 266.4 (1994): 805-828.